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## **MODEL REDUCTION OF PIEZOELECTRIC ENERGY HARVESTERS SUBJECT TO BAND-LIMITED, STOCHASTIC BASE EXCITATION**

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### **ABSTRACT**

In many scenarios where vibration energy harvesting can be utilized – particularly those involving bio-motions or environmental disturbances – energy sources are broadband and non-stationary. On the other hand, design procedures have been predominantly developed for harmonic or white noise excitation, specifically for single degree of freedom approximations of the transducer. In this paper, a general approach for design optimization of cantilevered, piezoelectric energy harvesters in the presence of band-limited, white-noise excitation is outlined. For this study, human and vehicular motions are considered; these complex waveforms are distilled into a small set of dominant features with regard to their impact on the power output of the device. Criteria based on modal participation factors, including pre-filtering of the disturbance, are used in guiding the reduction of the input and plant degrees of freedom in order to make the design optimization problem tractable. This process determines the error in assuming a low-order model for the transducer in the presence of broadband noise that may excite multiple modes of vibration. Furthermore, this study considers the quantitative impact of charge cancellation in higher modes and the benefits of inserting multiple electrodes along the length. To illustrate these methods, energy harvesters are designed for acceleration data collected from walking and car idling. It is shown that a simple method that is a generalization of naïve approaches that assume harmonic or white noise excitation and a single degree of freedom can determine which simplifications are appropriate and the inaccuracies that can be expected from them.

### **INTRODUCTION**

Ambient vibrations tend to be broadband and non-stationary in many practical scenarios for energy harvesting [1–3]. Until recently, much of the literature has focused on harmonic base excitation, which is only a crude approximation of real vibration sources. Very little analysis has been

performed on the broader class of stochastic base excitation, nor have any general design criteria emerged. There have been several experimental efforts that considered stochastic, broadband disturbances, with little analysis or predictive results [4–5], although the latter group has demonstrated their charge extraction circuit's insensitivity to changing excitation frequency.

The theory of random vibrations can be applied under limited conditions to the broadband energy harvesting problem. Halvorsen [6] has computed the optimal resistive loads assuming white noise, sinusoidal, and band-limited (i.e. band-pass filtered) excitation. The white noise case has been extended to the case of a resistor and inductor load model by Adhikari et al. [7]. Scruggs [8] has applied broadband regulation techniques to energy harvesters with active power electronics. This latter technique is similar to the linear quadratic regulator of modern control theory.

In this paper, a simple equation for estimating the power harvested from a broadband excitation source that excites multiple modes of the structure is derived based on a summation of the single mode expected power presented in [9]. This equation assumes that the excitation waveform can be written as the sum of harmonic components and that each component only excites one vibrational mode of the structure. Hence, the average (expected) power can be written as a sum of terms, each term representing the contribution of one of the excitation frequency components. This estimate for a single transducer excited by multiple frequencies is similar in form to the literal decoupling of frequency component contributions seen in multi-beam devices [10–12].

In the following sections, the modal equations of motion (EOMs) are briefly derived for a uniform bimorph beam. These are used to derive the general formula for the average (expected) power harvested from a stationary base excitation in terms of the frequency transfer functions for each vibration mode. A simplified form of the power equation is derived